Proceedings of the 2nd Annual World Conference
of the Society for Industrial And Systems Engineering
Las Vegas, NV, USA
November 5-7, 2013

Design of Barrel´s Cam Curve Profile Using B-Spline Curves

M López-Rodríguez, P Perez-Villanueva, and JP Nieto-Gonzalez

COMIMSA. Ciencia y Tecnología No. 790, Frac. Saltillo 400
C. P. 25290, Saltillo, Coahuila, México.

Corresponding author's Email: manuel.lopez@comimsa.com

Abstract: One of the biggest challenges the cam designer faces up is to find the equation that satisfies de timing diagram for a movement in a cam. There are several ways to get a model for a curve like Simple Harmonic Motion (SHM), Modified Trapezoidal Acceleration, Polynomial Functions, etc. This kind of equations works under some conditions of velocity and acceleration, but in the interval of its movement it can present erratic behavior because those equations are not allowed to control the discontinuities of acceleration or jerk. Getting forward in this problem this paper proposes the use of B-Spline curves to obtain the displacement function for a barrel cam, because its characteristics of local control give it an advantage over other families of curves. The methodology to build B-Splines curves is analyzed from the analytical point and then we use the software Dynacam to get the SVAJ diagram getting a shape of cam in real time. The results obtained were imported to Solid Works, reducing the time it takes to get a shape of a cam.

Keywords: B-Spline, Local Control, Jerk.

1. Introduction

From all mechanical elements, the cam is the most versatile one, because a simple movement can be transformed into a wide range of output movements (Norton RL, 2009). Nevertheless, one of the challenges when using a cam is that the velocities and accelerations can be very high, and that is why is necessary to take into account the right selection of the movement equation to achieve the best performance.

This work shows the literature review about the most used models for cam design, their characteristics and operation ranges. Also proposes the use of B-Spline curves to obtain the position function for a barrel cam, which own characteristics of local control in each point (knot) inside the curve’s motion.

2. Literature Review

According to Nguyen and Kim (2007) the cams are widely used in a number of applications in machines because you can obtain a countless quantity of movements from them. Even the follower cam system needs the cam profile being as smooth as possible so it can achieve the adequate cinematic and dynamic properties. There are many ways to mathematically express a cam profile. Among these functions that stand out for their movement are included the harmonic, the cycloidal, the modified harmonic, the trapezoidal curve, the polynomial, etc.

The spline curves have been applied in the cam design for over 20 years. The spline curves method is flexible enough to allow that movement programs refine and optimize them while the movement limitation can be satisfied. As they are, the spline methods have been applied to synthesized movements that would have turned complex with traditional methods. The processes to solve these problems include the task to satisfy a set of arbitrary movement restrictions.

In other hand, the objective was to address with several design factors to cinematic and dynamic optimization. But the fact that is based in B-Spline curves the affectation only takes a part of the points close to it. According to this experiment results, the analysis could show some profile points that are not desirable specially when these adjusts are required in the speed or acceleration curves.

2.1 Dynamic Analysis of the Mechanism

Livija (2006) mentions two cam mechanism models that are used for their dynamic analysis. A model considers a rigid cam, the shaft that drives the cam and its follower is considered an elastic body (spring). The other model considers the cam as an elastic body as well as the driven shaft, and the follower as a rigid element.

For the first model, the system resumes a simple system mass-spring of one degree freedom. The mathematic model is a parametric differential equation of a second order that provides always a pre-charge in the way that the follower always maintains contact with the cam at the specified speed.

When the cam performs its rotation, this moves the follower. In a subsequent way, the follower is shifted. The inconsistence of modeling a cam system by this method in order to find a satisfactory solution is that by nature it is impractical to apply it due to the simplicity of the model.

With a high load and higher velocities, the vibration phenomenon in the cam shaft can be seen, and then the effect of the cam elasticity should be taken into account to model the effect of the elasticity of its joints. This results in a non-lineal differential equation with time variation. The effect in the cam is a variation in the torque due to the effect of the follower that induces torsional parametric vibrations and self induced in the cam shaft that affect its movement and its contact force.

2.2 Software use to Obtain Cam Profiles

Jianping and Zhaoping (2011) use "Pro-engineer" software and its "datum-Graph function" characteristic, which creates the displacement graph for the follower and obtains the displacement curve according to the type of movement needed; it also controls the profile for the movement of the cam, by a section sweep in the software.

Wu et al (2007) focuses its study comparing the cam mechanisms with tensors and the tensor bar mechanisms only. Their study shows that the mechanisms that use cams with tensors observe superior dynamic properties comparing to the properties of the second mechanism. The opportunity area is that this theory is only applicable in mechanisms that act under low velocities, because these can show mistaken behaviors in their designed movement profile comparing to the real following error.

3. Procedure

The following list represents the methodology to follow for the proposed model.

1. Position, speed, acceleration and jerk analysis for the Oscar Roll cam.
2. Define the time diagram for STK001 cam.
3. Modeling the objective function with B-Splines.
4. To obtain the cylindrical cam profile that satisfies the speed restriction of 1500 ppm, without discontinuity in the cam movement interval.
5. To evaluate the results and validate the proposed model.
6. Developing the curve points for use in the Solid Works software.

4. Model Formulation

A B-spline curve is defined as

Objective function:

$$\boldsymbol{P}(\boldsymbol{t}) = \sum_{i=1}^{n} P_i \, \mathrm{N_{i,k}}(\mathrm{t}) \quad (1)$$

Where:

$\{\mathrm{P_i}: \mathrm{i} = 0,1, \ldots., \mathrm{n}\}$ Are the control points

k is the order of the polynomial segments of the B-Spline curve. The order k means that the curve is made from polynomial parts of $\mathrm{k} - 1$ grade.

$\mathrm{N_{i,k}}(\mathrm{t})$ are the B-Splines function of folding that are normalized. These follow the k order of real numbers $\{\mathrm{t_i}: \mathrm{i} = 0,1, \ldots., \mathrm{n} + \mathrm{k}\}$, that are commonly known as the knots sequence. They can be described in the following way.

$$N_{i,1}(t) = \left\{\begin{cases} 1\ if\ u\ E\ \ [t_i, t_{i+1}) \\ \ \ 0\ otherwise \end{cases}\right. \quad (2)$$

If $k > 1$,

$$N_{i,k}(t) = \frac{t - t_i}{t_{i+k-1} - t_i} N_{i,k-1}(t) + \frac{t - t_i}{t_{i+k-1} - t_i} N_{i,k-1}(t) \quad (3)$$

With B splines we have

$$S(\theta) = c_1 B_{m,1}(\theta) + c_2 B_{m,2}(\theta) + \cdots + c_n B_{m,n}(\theta) \quad (4)$$

Its differential can be represented in the following way:

$$s'(\theta) = (m-1)\sum_{n=i-m+2}^{j-1} \frac{c_n - c_{n-1}}{\theta_{n+m-1} - \theta_n} B_{m-1,n}(\theta) \quad (5)$$

Where the B's are called B-Splines of order m. These B-Splines will take the role of the exponential in the polynomial function and the displacement design can be obtained in the same way that the polynomial curve; the coefficients can be determined solving the equations of the frontier equation and the interpolation points.

5. Application and Rresults

Kimberly Clark, Ramos Arizpe plant manufactures diapers for the national market. Continuously this enterprise has been trying to reduce the number of transversal shrinks in the diapers which are related to the quality image of the product. These shrinks are originated mostly for the impact of the diaper in a blade folder against the chain of the stacker paddles that are shown in Fig 1.

Entrance to stacker Impact against stacker

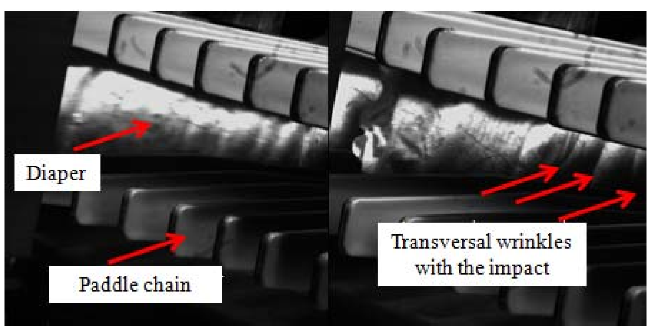

Figure 1. Diaper's Stacker

According to the literature review it is possible to establish a process to obtain a profile for a cylindrical cam using B-Spline curves and the Dynacam software for this particular problem. The goal is that the cam design takes the diaper before it accelerates to stack it with a lower acceleration. With the use of B-Spline curves we can obtain curves for the cams which can be modified at any point of the interval and observe a good performance in acceleration and jerk. In order to solve this problem an existent mechanism is used as a base; this mechanism uses a cylindrical cam, which applies one component to the diaper at a low speed and then accelerates to finish its movement. The figures 2-4 shows different values for torque, speed and acceleration of the cam for the different operation speeds (DPM: diapers per minute) obtained using the movement control software RSLogix 5000.

	Average speed Oscar Roll´s cam	16 mm/seg
	Actual Speed Oscar Roll´s cam	16 mm/seg
	Actual Acceleration Oscar Roll´s cam	2 mm/seg ^ 2
	Torque feedback Oscar Roll´s cam	10% Nominal
	Actual position Oscar Roll´s cam	
	Line Speed in DPM	950 DPM

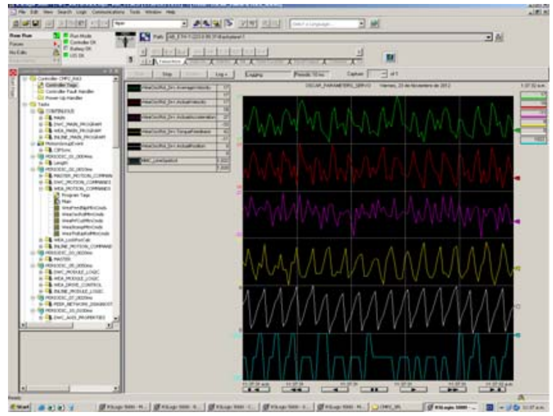

Figure 2. Speed and acceleration trend for the Oscar Roll Cam at 950 DPM

	Average speed Oscar Roll´s cam	16 mm/seg
	Actual Speed Oscar Roll´s cam	16 mm/seg
	Actual Acceleration Oscar Roll´s cam	- 11 mm/seg ^ 2
	Torque feedback Oscar Roll´s cam	32% Nominal
	Actual position Oscar Roll´s cam	
	Line Speed in DPM	1000 DPM

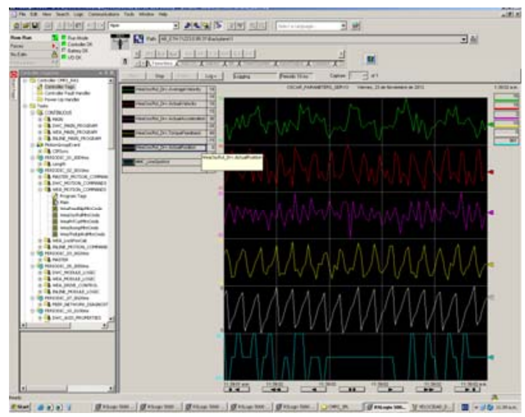

Figure 3. Speed and acceleration trend for the Oscar Roll Cam at 1000 DPM

	Average speed Oscar Roll´s cam	17 mm/seg
	Actual Speed Oscar Roll´s cam	16 mm/seg
	Actual Acceleration Oscar Roll´s cam	- 11 mm/seg ^ 2
	Torque feedback Oscar Roll´s cam	48% Nominal
	Actual position Oscar Roll´s cam	
	Line Speed in DPM	1100 DPM

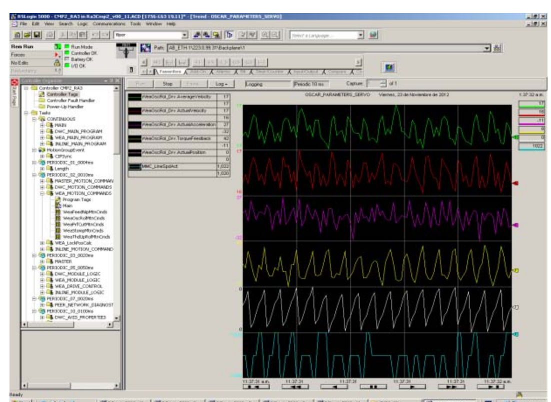

Figure 4. Speed and acceleration trend for the Oscar Roll Cam at 1100 DPM

Time diagram for the proposed cam

The proposed cam time diagram for the stacker is represented in figure 5. The input data are the following:

- The S2 Ultraconfort product has a total length of 12 in, if the required machine speed is 1500 Diapers per minute (DPM) (25 diapers per second), the lineal velocity will be 40 in/sec.

- Accelerate the follower from zero to 480 in/sec
- To maintain a velocity of 480 in/sec for 20ms
- Decelerate the follower to cero velocity without passing 0.25in from the start position
- Cycle time of exactly 40ms

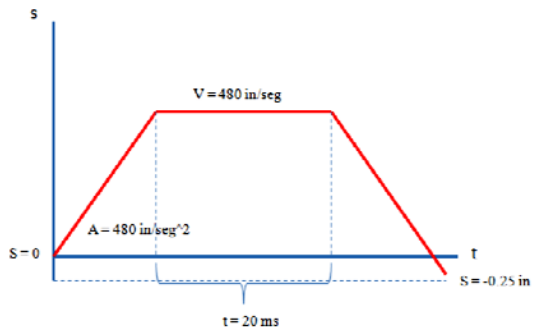

Fig 5. Timing diagram for STK001 cam

Boundary conditions for the proposed time diagram:

Angular velocity= $\frac{9.6}{\pi}$
$c_1 = 9.6\ in$
$c_8 = 0\ in$

By equation (5) we have:

$$\frac{9.6}{180} = c_2 - c_1$$

The equations lead to:
$c_2 - c_1 = 0.0533333$
$c_1 - 2c_2 + c_3 = 0$
$c_8 - c_7 = 0.0533333$
$c_6 - 2c_7 + c_8 = 0$

Solving our equation system the coefficients are:
$c_1 = 9.6$
$c_2 = 9.6533333$
$c_3 = 9.7066666$
$c_6 = -0.1066666$
$c_7 = -0.053333$
$c_8 = 0$

To obtain c_4 and c_5 the equations are fixed and for equation (5) the maximum displacement is:
$9.907 + 0.0277c_4 + 0.00123c_5 = 9.85$
$0.654 + 0.310059c_4 + 0.01956c_5 = 0$
$c_4 = -1.9166$
$c_5 = -3.0542$

The curve equation that satisfies the time diagram is:

$$s(\theta) = 9.6\ B_{6,1}(\theta) + 9.6533333B_{6,2}(\theta) + 9.7066666B_{6,3}(\theta) - 1.9166B_{6,4}(\theta) - 3.0542B_{6,5}(\theta) - 0.1066666B_{6,6}(\theta) - 0.053333B_{6,7}(\theta)$$

The figures 6 - 7 illustrate the SVAJ diagram and the contour for the model proposed above in Dynacam software.

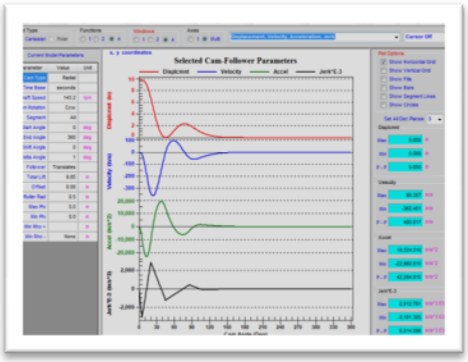

Fig 6. SVAJ diagram for the cam profile

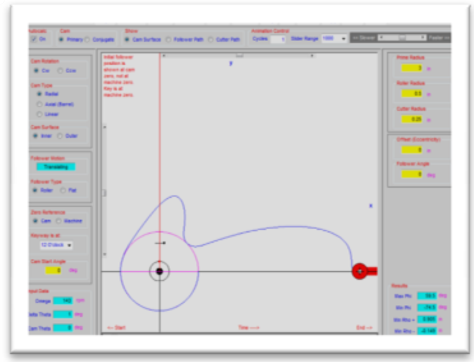

Fig 7. Profile contour movement of the STK001 cam

6. Conclusion

The present work shows how the B-Splines curves methodology can be applied to the selection of the displacement curves for the proposed time diagram for the case study.

When applying the B-Splines method, it can be deduced:

- The equation grade for cam design should be of the order of 6, because an equation of a lower degree would cause discontinuities in the acceleration or jerk. By other hand if we use an equation with exponents higher than 6, the curve moves away from the control point, so the curve would not be the best description for the desired curve for the cam profile.
- Because this is an iterative method and has the need to solve as many equations as unknown quantity for the control points, the use of software to solve these equations is recommended.
- With the use of cam profile creation tools the designer can see in real time the forms of the cams because modifications can be introduced in the software and even see contact parts.

For future work the optimal control points should be obtained by an optimization algorithm to find the cam equation and this means lower design and calculation time.

7. References

Cveticanin, L. (2007). Stability of motion of the cam–follower system. *Mechanism and Machine Theory, 42*, pp. 1238–1250.

Jianping, S. and Zhaoping, T. (2011). The Parametric Design and Motion Analysis about Line Translating Tip Follower Cam Mechanism Based on Model Datum Graph. *Procedia Engineering 23*, pp. 439 – 444.

Nguyen, V.T. and Kim, D-J. (2007). Flexible cam profile synthesis method using smoothing spline curves. *Mechanism and Machine Theory.* 42, pp. 825–838.

Norton R.L. (2009). *Cam Design and Manufacturing Handbook 2nd. Edition*. Industrial Press, Inc. USA.

Wu, L-I., Chang, W-T., and Liu, C-H. (2007). The design of varying-velocity translating cam mechanisms. *Mechanism and Machine Theory, 42*, pp. 352-364.